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Probability Theory as Applied to Reliable Network Design

James D. LaRue

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Unclassified Report

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results. The reliability of this network is calculated and compared to several other designs.

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2. Redundant networks
3. Quad Networks

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FOREWORD

This report was prepared by James D. LaRue of the Guidance Section, Guidance and Control Branch, Systems Analysis Division, Directorate of Advanced Systems Planning, Deputy for Technology, Aeronautical Systems Division, Wright-Patterson AFB, Ohio. The work was accomplished under Project No. 5401, "Advanced Systems Synthesis and Analysis Methods."

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ABSTRACT

The probability of success equations for redundant networks are derived from basic component failure statistics. The components have two modes of failure, open or short. The report calculates the equations for four basic redundant networks. A network is then designed based upon these results. The reliability of this network is calculated and compared to several other designs.

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DEFINITIONS

Density Function, $f(t)$ - The probability that an event of a given random variable will occur, expressed as a function of that variable.

Disjoint - Two events are disjoint if the occurrence of one excludes the occurrence of the other.

Distribution Function, $F(t)$ - The probability that in a random event, the random variable is not greater than some designated value.

Failure - The event of unsatisfactory response of a component or device. The failure of a resistor can be specified quite easily whereas the failure of a computer is quite difficult to specify due to the large number of possible modes of unsatisfactory response.

Failure Rate - The ratio of the probability that failure occurs in the interval, given that it has not occurred prior to t, the start of the interval, divided by the interval length. The formula for failure rate is

$$\frac{P(t) - P(t + \Delta t)}{\Delta t \cdot P(t)}$$

Hazard Rate, $z(t)$ - Also called instantaneous failure rate. Defined as the limit of the failure rate as the interval length approaches zero.

$$z(t) = \lim_{\Delta t \to 0} \frac{P(t) - P(t + \Delta t)}{\Delta t \cdot P(t)} = \frac{f(t)}{P(t)}$$

Mean time between failures, MTBF - The arithmetic average of the time to failure of all the items considered. This mean value has meaning only when the distribution function of the failures is known.

Probability of Success, $P(s)$ - The ratio of favorable events to total events.

Random Process - An ensemble of time functions $\{k_{X(t)}\}$.
$-\infty < t < \infty$, $k = 1, 2, 3 \ldots$ such that the ensemble can be characterized through statistical properties.

Redundancy - The condition which exists if an element can fail in a network and the network continues to function. If elements are switched in and out of the circuit the redundancy is known as passive. If the elements are energized until failure, the redundancy is known as active.

Reliability, $P(t)$ - The probability that a device will operate successfully for a given period of time when operated under specified conditions.

INTRODUCTION

A method of applying probability theory to design procedures is presented. An increasing percentage of electronic equipment designed today must meet some standard of satisfactory operation. The design engineer must design his equipment to reach or exceed a definite reliability goal. This goal can be a specified mean time to failure or a probability of successfully operating a given number of hours. It is the intent of this report to review the current accepted techniques in predicating reliability of electronic equipment and then use probability theory to design networks which are extremely reliable as compared to conventionally designed networks.

CURRENT TECHNIQUES

Reliability is a relatively new science. The first concerted effort to gather failure data was started early in 1950. By 1955 prediction techniques were formulated using the statistics generated by the collection of failure data. B. Epstein of Wayne University and M. Sobel of Bell Telephone Laboratories did much of the early sifting of these failure statistics. Arinc Research Corporation acting on behalf of commercial airlines was very active in these early investigations. It was already known by 1950 that the times of wear out failure followed a normal distribution. A great amount of information is implied in the statement "times of wear out failure followed a normal distribution." The length of life of an incandescence light bulb is a random variable. The standard method of describing a random variable such as bulb life is to state the probability of the light bulb reaching or exceeding some value of bulb life. In other words, what is the probability that a given 50 watt light bulb will operate at least 500 hours when operated under standard conditions. The random variable is bulb life; the random event is the failure of the bulb to work? If a large number of bulbs are placed on life test, time of the failure being recorded, the mean life and dispersion around the mean can be calculated. The probability of a 50 watt bulb reaching 500 hours is then the ratio of bulbs operating at 500 hours to the total number starting the test. The plot of probability of reaching some value of time as a function of time is known as the cumulative distribution function. The word "normal" describes a particular shape to this plot. Bearings, clutches, motor brushes, internal combustion engines all have failure records which support the wear out - normal distribution relationship. Wear out failures are not the only type of failures to occur. For example, with car tires one type of failure is the wearing off of tread. A second type of failure is caused by road hazards. If a tire runs over a road hazard, a blow-out occurs. The first type of failure follows a normal distribution with the random variable being mileage rather than time. The second type of failure is characterized by having a constant risk. The constant risk implies that there are a fixed number of road hazards per 1000 miles. This number of hazards is not a function of the number of miles driven. A random event which is associated with a constant risk can be shown to possess an exponential probability function.

But resistors, capacitors, semiconductors, and other electronic components do not wear out in the accepted sense that something is worn or eroded away. Yet random failures of these components do occur.

Sobel and Epstein have written several papers dealing with the statistics of life testing. The random variable recorded during life testing is the time of failure of the device.

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When electronic equipment is placed in life test under the environment for which it was designed, the time of failure is distributed in accordance with the exponential probability distribution function.

Figure 1 illustrates the normal distribution and the exponential distribution as well as illustrating values of terms commonly used in reliability. For both the normal and the exponential life test 90 items were placed on test.

The data for figure 1 is not from a real test; therefore, the resulting curves are more nearly perfect than would be expected from actual life tests. The number of failures which occurred during each hour of operation is given. Failure rate is the ratio of units failed during the hour interval to the number of units entering the hour. The sum of survival rate and failure rate is unity. Probability of survival is the ratio of units surviving the hour to the total number of units starting the test.

Before examing the failure data of several electronic systems the method of validating the postulated distribution of the random failures must be explained. One method of accepting or rejecting the postulated distribution is known as the Kolmogorov-Smirnov one sample test. The method involves tabulating the cumulative distribution functions, survival curve figure 1, of the actual failure data and the postulated cumulative distribution function. It can be shown that the absolute difference between the two functions is itself a random variable with a known distribution. The method is called one sample because the maximum absolute difference is calculated and serves as a criteria for acceptance or rejection of the postulated distribution. The maximum absolute difference is compared to a table of critical values of difference in the Kolmogorov-Smirnov one sample test. If for a given sample size and level of significance the maximum absolute difference is less than the tabular value, the postulated distribution is the best fit that can be had. The level of significance is an indication of the error in accepting the postulated distribution when the postulated distribution should be rejected. The usual value for this is 0.05 or 5 times out of 100 the postulated distribution will be accepted when it should be rejected.

The Radio Corporation of America in conjunction with the Air Force has collected failure data on hundreds of ground based electronic systems. Radar, navigation, and communication equipment were included in this study. Table 2 is a Kolmogorov-Smirnov one sample test for an AN/FPS-3, ground based radar. The main points of interest are:

(1) Large number of failure, 116
(2) Mean life 55 hours, 6409/116 = 55 hr.
(3) The exponential distribution fits the data.

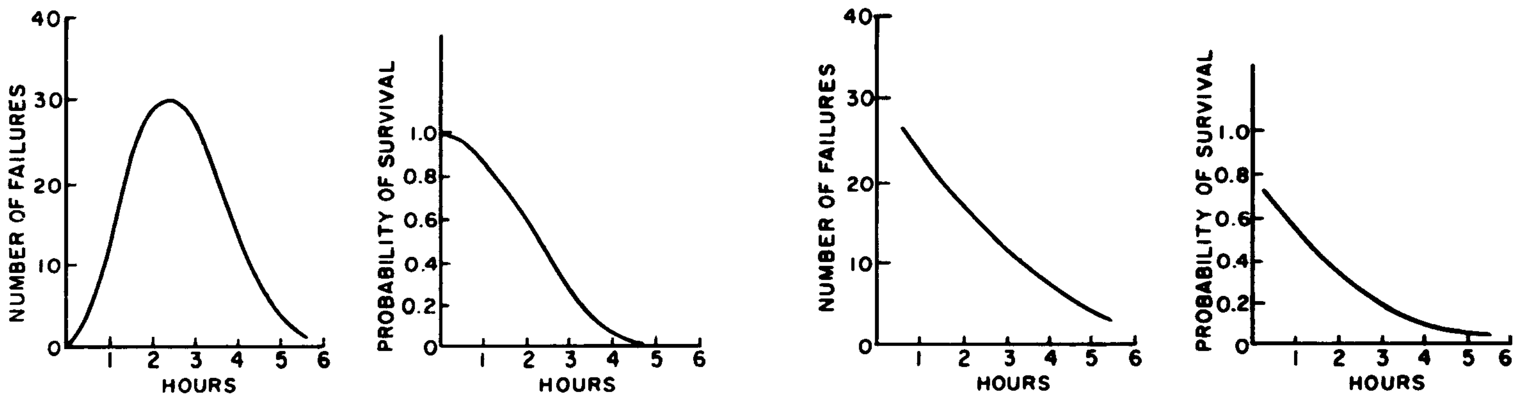
Figure 2 is a plot of probability of success versus operate time. The exponential is very close to the actual data. Arinc Research Corporation investigated the AN/APS-20E airborne radar. Their objective was to modify the radar such that the radar becomes more reliable. Figure 3 is a plot of four curves, two exponential and two observed. The unmodified and the modified systems are seen to follow an exponential distribution.

Figure 4 is a plot of probability of success for a vacuum tube, 2D21W. The basic exponential probability of success is very closely approximated. This data came from reliability studies of electronic equipment on the USS Forestal during 1958 and 1959.

The concept of hazard or risk was discussed with failure modes of tires. The mathematical definition of hazard is "The probability that a failure will occur in the next instant of time, assuming previous survival." Using this definition equations can be

TABLE I
TIMES OF FAILURE AND ASSOCIATED RATES

NORMAL DISTRIBUTION					EXPONENTIAL DISTRIBUTION				
TIME	NO. OF FAILURES	FAILURE RATE	SURVIVAL RATE	PROBABILITY OF SURVIVAL	TIME	NO. OF FAILURES	FAILURE RATE	SURVIVAL RATE	PROBABILITY OF SURVIVAL
0 – 1	4	.0445	.9555	.9555	0 – 1	30	.333	.667	.667
1 – 2	21	.2440	.7560	.7222	1 – 2	20	.333	.667	.444
2 – 3	30	.46203	.5380	.3889	2 – 3	14	.350	.650	.289
3 – 4	25	.7140	.2860	.1111	3 – 4	9	.346	.654	.189
4 – 5	8	.8000	.2000	.0222	4 – 5	6	.353	.647	.122
5 – 6	2	1.0000	---	---	5 – 6	4	.364	.636	.078



NORMAL DISTRIBUTION OF FAILURES
NORMAL SURVIVAL CURVE
EXPONENTIAL DISTRIBUTION OF FAILURE
EXPONENTIAL DISTRIBUTION CURVE

Figure 1. Probability Distributions Used in Reliability

developed which relate the hazard equation to the reliability function. If the hazard is known to have some function of time, the reliability as a function of time becomes specified. Several component manufacturers have investigated the hazard function of their components. The following equations show the relation between a constant hazard and the exponential reliability function. If a component manufacturer can show a reasonably constant hazard, then the assumption of an exponential failure rate is valid. In reliability literature the term failure rate is utilized rather than hazard. Throughout this report hazard and failure rate will be used interchangeably although failure rate is an incorrect (mathematically speaking) concept.

$P_s(t)$ - reliability function - probability of success

$P_F(t)$ - unreliability function - probability of failure

$$P_s(t) + P_F(t) = 1 \tag{1}$$

$$\frac{dP_s(t)}{dt} = f(t) \text{ - probability density function} \tag{2}$$

$f(t_1)\,dt$ - probability of failure in the interval dt centered at t_1

$\lambda(t)$ - hazard or failure rate

Hazard is a conditional probability function as it is a function of the probability of operating to a given time coupled with the probability of working in the next interval. The relationship between hazard, probability density function, and the reliability function can be developed by the use of conditional probability theory. This approach is very erudite and not in line with the intent of this work. Therefore, rather than including several pages of abstract mathematics showing the relationship between hazard, probability density function, and reliability function, it will be defined as the ratio of the probability density function to the reliability function.

$$\lambda(t) = \frac{f(t)}{P_s(t)} \tag{3}$$

If the hazard is constant, then equation (3) can be written as:

$$\lambda(t) = \frac{1}{P_s(t)} \cdot \frac{d\left[1 - P_s(t)\right]}{dt} \tag{4}$$

$$\lambda dt = \frac{-1}{P(t)} \cdot d\left[P(t)\right]$$

$$\lambda t = -\ln P_s(t)$$

$$P_s(t) = e^{-\lambda t} \tag{5}$$

Where $P_s(t)$ is the probability of success as a function of time when the hazard is constant.

The mean value, μ, of a random variable is the limit of the sum of the assumed values when each value is multiplied by its appropriate probability of occurrence.

TABLE 2

OBSERVED AND THEORETICAL RELIABILITY FUNCTIONS FOR AN/FPS-3

TIME OF TEST - 6409 HOURS
MEAN LIFE (m) - 55 HOURS

TRUE FAILURES - 116
d(max)(0.05) = 0.126

PERIOD	CUMULATIVE FAILURES	CUMULATIVE PROBABILITY	THEORETICAL CUMULATIVE PROBABILITY	ABSOLUTE VALUE p(o) - p(e)
t (hr)	O	p(o)	$p(e) = e^{-t/m}$	d
0 or more	116	1.000	1.000	.000
10 or more	96	.828	.834	.006
20 or more	77	.664	.695	.031
30 or more	61	.526	.580	.054
40 or more	53	.457	.484	.027
50 or more	45	.388	.404	.016
60 or more	38	.328	.336	.008
70 or more	29	.250	.281	.031
80 or more	21	.181	.235	.054-d(max)
90 or more	18	.155	.194	.039
100 or more	15	.129	.162	.033
110 or more	13	.112	.135	.023
120 or more	11	.095	.113	.018
130 or more	10	.086	.094	.008
140 or more	10	.086	.078	.008
150 or more	10	.086	.065	.021
160 or more	8	.069	.054	.015
170 or more	8	.069	.046	.023
180 or more	7	.060	.038	.022
190 or more	6	.052	.032	.020
200 or more	3	.026	.026	.000

d(max) = 0.054 d(max)(0.05) = 0.126

Therefore, the observed cumulative probability fits the theoretical cumulative probability distribution at a 5 percent level of significance.

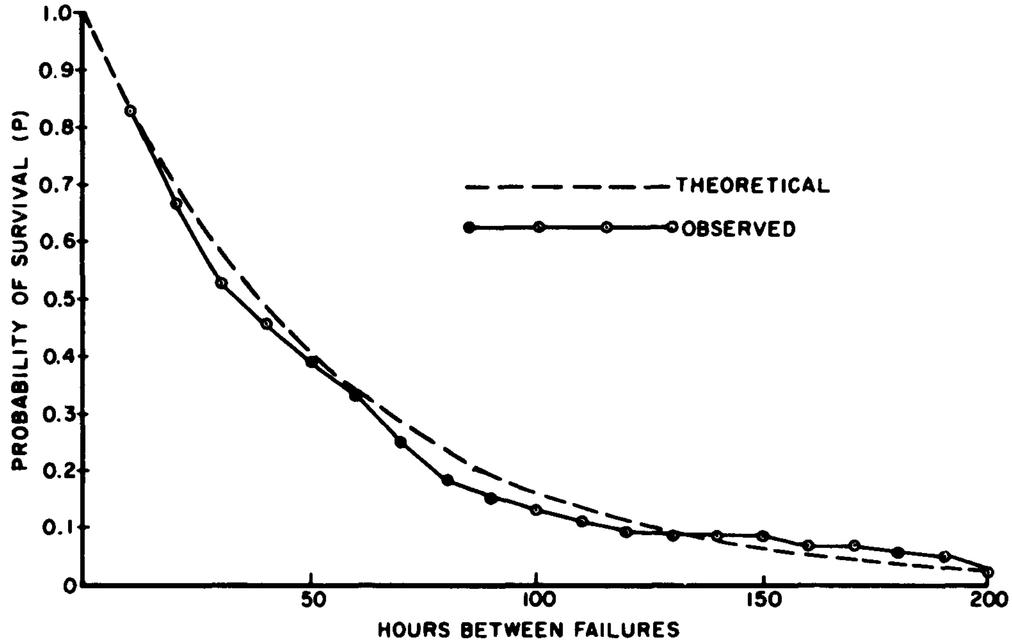


Figure 2. Probability of Survival Curves for AN/FPS-3

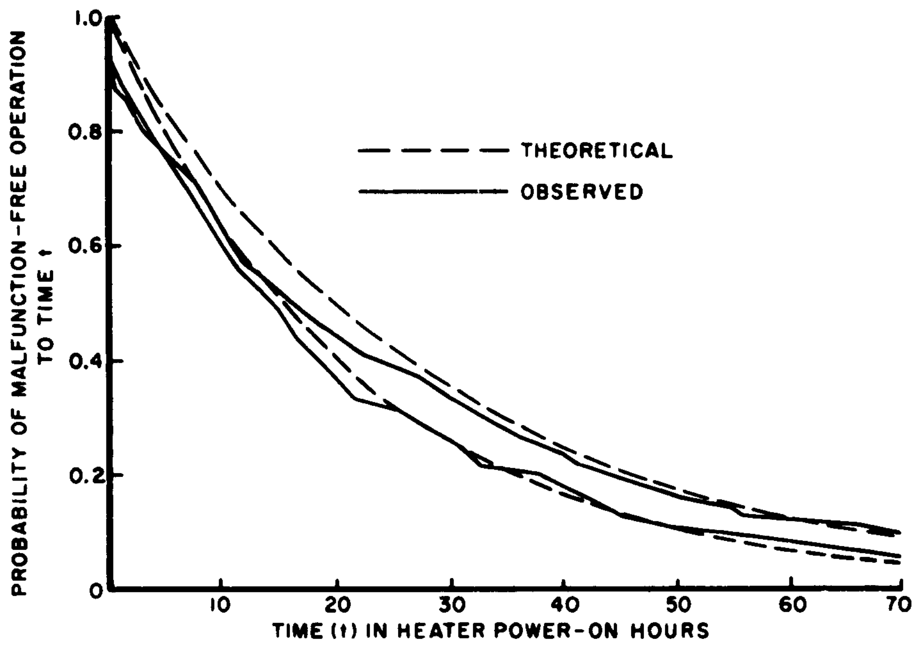


Figure 3. Theoretical and Observed Reliability Functions for Unmodified and Modified AN/APS-20E Systems

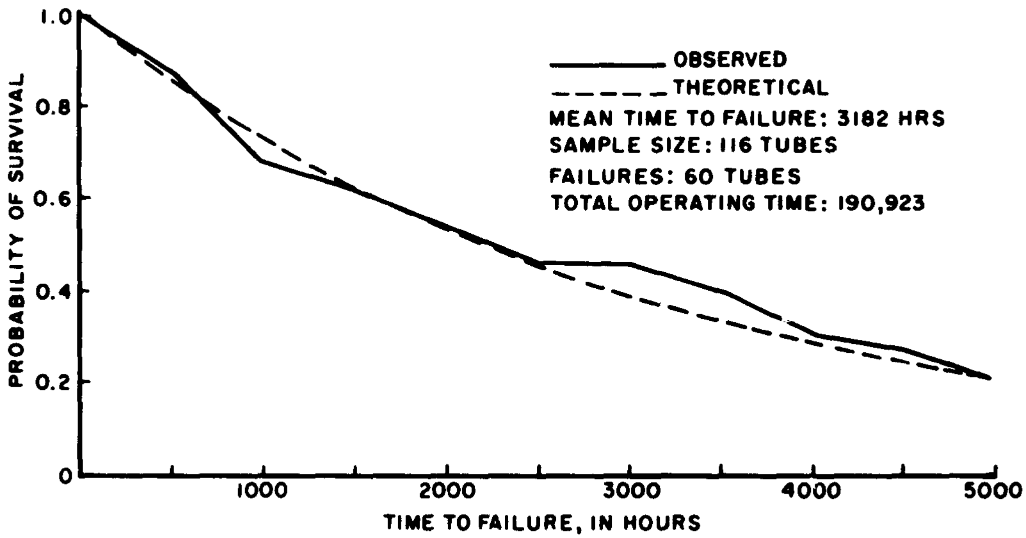


Figure 4. JAN-5727/2D21W Reliability Functions

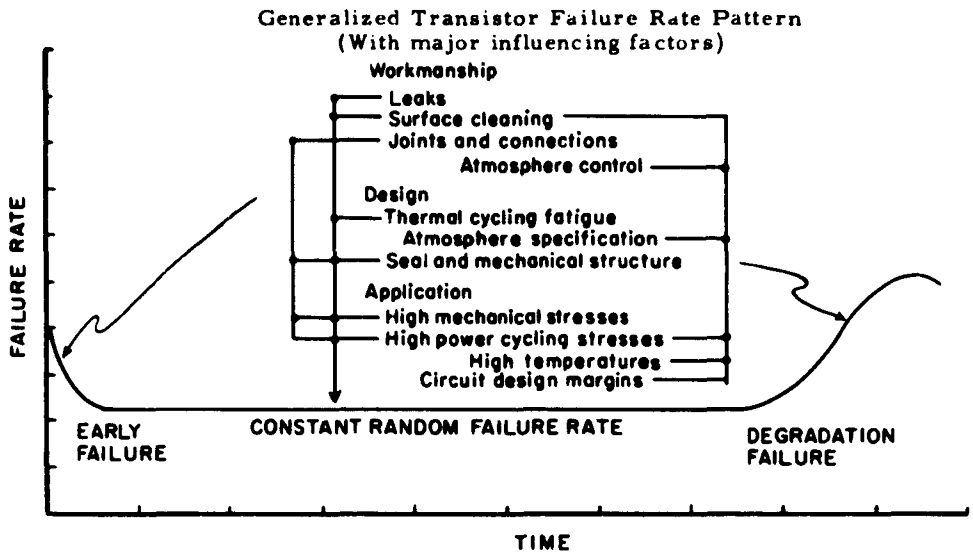


Figure 5. Transistor Failure Rate (General Electric)

$$\mu = \int_0^{\infty} t \cdot f(t)\, dt$$

$$= \int_0^{\infty} t \left(\frac{-e^{-\lambda t}}{\lambda}\right) dt$$

$$\mu = \frac{1}{\lambda} \tag{6}$$

The mean value is also the arithmentic average, $\overline{\overline{T}}$, of the random events. This arithmetic average is known as the mean time before failure, MTBF,

$$\mu = \overline{\overline{T}} = \frac{1}{\lambda}$$

$$e^{-\lambda t} = e^{-t/\overline{\overline{T}}} \tag{7}$$

This identity is used throughout the report.

Figure 5 presents the causes of failure which are present during the life of a semiconductor. The curve is often called the bath tub curve for obvious reasons. The curve represents the experience General Electric engineers have had in the production and life testing of semiconductors.

Figure 6 is a plot of failure rate for several Philco transistors. Figure 7 is a plot of the Kolmogorov-Smirnov test for exponentiality of the 2N496-2N495 2N1118-2N119. If the stepped line crosses the UCL, the hypothesis of exponential failure is rejected. This data came from a Philco report entitled "Reliability Report on the Philco SAT Transistors" dated 1960.

The strong belief in components possessing an exponential distribution is also based upon the serial nature of most electronic networks. The serial nature concept has one basic assumption; all components must perform satisfactorily or the system fails. Probability theory dictates that under these conditions the probability of success of the overall system is the product of the individual component probabilities of success.

$$P\text{ (system)} = \prod_{i=1}^{N} P_i \text{ (component)} \tag{8}$$

Ample data substantiates that the system probability of success is exponential. The only mathematical form which P_i can be is a constant or an exponential.

Since the system probability of success is the product of the individual probabilities of success the system failure rate is the sum of the individual failure rates assuming an exponential distribution.

$$e^{-\lambda_T t} = e^{-\lambda_1 t} \cdot e^{-\lambda_2 t} \cdots e^{-\lambda_N t} \tag{9}$$

$$e^{-\lambda_T t} = e^{-\sum_{i=1}^{N} \lambda_i t} \tag{10}$$

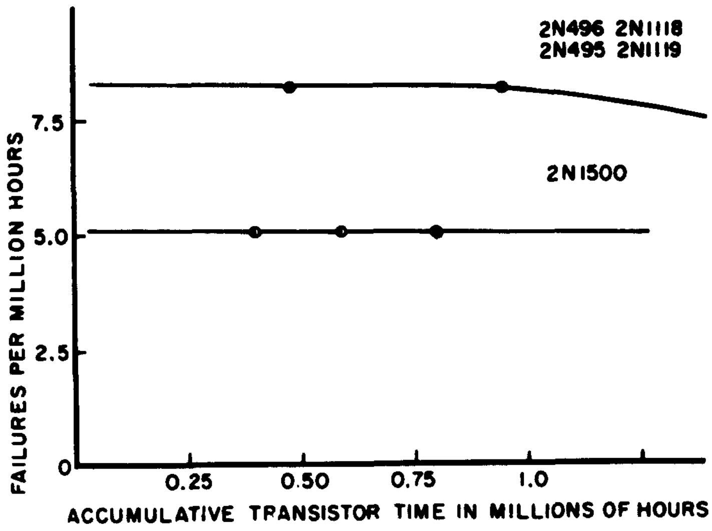


Figure 6. Transistor Failure Rate (Philco)

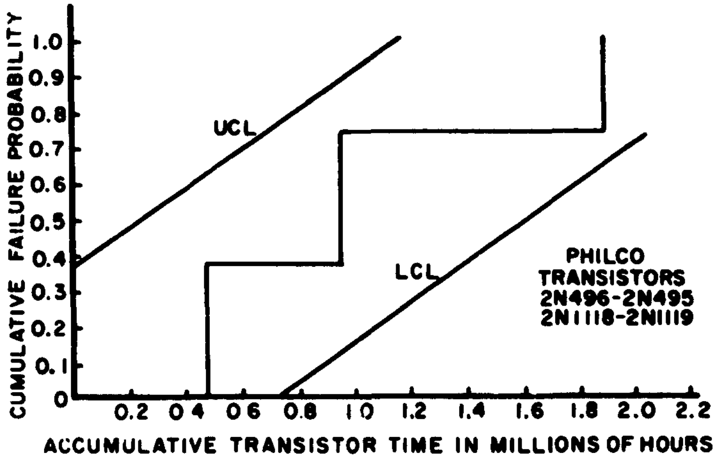


Figure 7. Kolmogorov-Smirnov Test for Exponentiality

where λ_T - system failure rate

λ_N - Nth component failure rate

The failure rate of each type of component is obtained either from the component manufacturer, life test data, field data or from any of the many tabulated listings now available. One such source is a report by Arinc Research Corporation entitled "A Reliability Study of Microwave and Transmitting Tubes, Semiconductors, Relays and Other Parts," dated September 30, 1960.

What ultimately determines the validity of the exponential distribution as a reliability predictor, that is the summing of individual failure rates to form system failure rate, is the designing of a complex electronic device to a definite reliability goal. In 1959 the Radio Corporation of America agreed to build for the Air Force an airborne data link which had to possess a minimum acceptable mean time to failure of 200 hours. At the end of 10,120 hours of accumulated test hours the calculated mean time to failure was 211 hours.

PURPOSE

Suppose the design engineer is given the task of designing an electric network which must exhibit a minimum acceptable probability of success. This implies that the network's failure rate, summed up from all components, must be below some value. For purposes of discussion the design engineer is using failure rate data which is up to date and accurately represents the components he will use. The problem arises when the summed failure rate is greater than the acceptable failure rate.

In the last several years this problem has arisen and has been solved by using passive redundancy. In redundant design those subsystems which have a high failure rate are either placed in parallel or series and switched on and off as needed. Thus, early effort in redundancy centered upon complete assemblies, for example two complete photo electric tubes and associated circuitry were used in star trackers. If phototube A failed, then phototube B was switched on to operation. Generators and motors fall into this category.

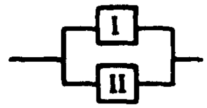
This technique works for certain items; however, in an airborne digital computer which employs 8,000 transistors, 24,000 resistors, 32,000 diodes, 7,000 capacitors none stand out as critical or frequent failures. Should the entire computer be duplicated and carried as a spare? Or should the individual resistors, capacitors, transistors, and others be made redundant so that failures of individual components will not cause network failure?

The present accepted method of analyzing the gains in reliability due to redundancy considers all failures are identical. This was adequate for passive redundancy where the failed element is switched out of the circuit. In active redundancy failed elements are not switched out, but remain in the circuit. It is quite clear that the mode of failure of an actively redundanded element affects the operation of the network. For example, paralleling of diodes is one example of active redundancy. If a diode shorts, then the net has failed; however, an open diode still permits network operation. In this report redundant networks are examined and the probability of success equations are derived on the basis that some elements will fail in an open or short.

REDUNDANT NETWORKS

Probability of Success Equations

The usual form of redundancy consists of two or more identical elements arranged in a parallel or series configuration. If the mode of failure is a short, then two elements in series constitute redundancy. If the mode of failure is an open, then two elements in parallel constitute redundancy.



q_s -probability of short

q_o -probability of open

Figure 8 - Parallel Network

$P_s = 1 - q_s - q_o$ probability of success of each element. Two arithmetic operations used frequently in probability are P(A + B) and P(A,B). P(A + B) denotes the probability that either A or B or both occur. P(A,B) denotes the joint probability that both A and B occur.

$$P(A + B) = P(A) + P(B) - P(A,B) \tag{11}$$

$$P(A,B) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B) \tag{12}$$

P(B/A) is the probability of B occurring, given that A has occurred.

Two events are said to be mutually exclusive if

$$P(A,B) = 0 \tag{13}$$

Thus

$$P(A + B) = P(A) + P(B) \tag{14}$$

Two such events are mutually independent if

$$P(A,B) = P(A) \,.\, P(B) \tag{15}$$

By definition $q_s \,.\, q_o = 0$ since one element cannot open or short.

To determine the probability of success or its equivalent the probability of failure of any network, the problem is to form a list of all events which will cause the network to perform successfully or unsuccessfully depending, of course, upon which probability is being formulated. If both probabilities are formulated, then none of the terms in one list will appear in the other. Mathematically these lists or sets are said to be mutually exclusive. The individual lists must comprise all the events leading to the particular mode being formulated. If a list is complete, then the list is said to be exhaustive. The term disjoint is used interchangeably with mutually exclusive. It may be easier to form an exhaustive set of disjoint events comprising successful operation or it may be simpler to operate on the failure events. With the parallel configuration it is easier to work with failures since two mutually exclusive events form an exhaustive set of failures.

Let α represent the probability that element I or II shorts

Let β represent the probability that both element I and II opens.

Then the probability of failure, P_F, of figure 8 is

$$P_F = \alpha + \beta \tag{16}$$

$$\alpha = q_s + q_s - q_s^2 \tag{17}$$

$$\beta = q_o \cdot q_o \tag{18}$$

$$P_F = 2\, q_s - q_s^2 + q_o^2 \tag{19}$$

$$P_S = (1 - q_s)^2 - q_o^2 \tag{20}$$

The same approach is used for figure 9.

Figure 9 - Series Network

The exhaustive set of disjoint failures is arrived at as follows:

α is the probability that element I or II opens.

β is the probability that element I and element II shorts

$$P_F = \alpha + \beta \tag{21}$$

$$\alpha = 2\, q_o - q_o^2 \tag{22}$$

$$\beta = q_s^2 \tag{23}$$

$$P_S = (1 - q_o) - q_s^2 \tag{24}$$

The series parallel network presents a similar problem.

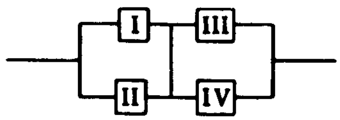


Figure 10 - Series Parallel Network

It can be shown that two mutually exclusive failure events can form an exhaustive set, and the problem may be solved in this manner. A far simpler method is to regard figure 10 as a series network such as figure 11.

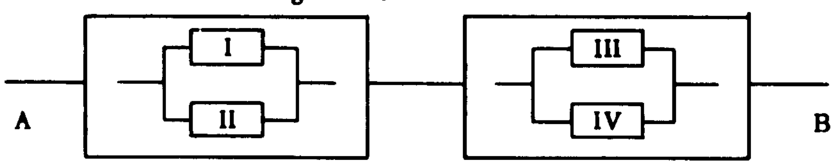


Figure 11 - Serial Representation

Now q_s of unit A is as follows

$$q_s\,(A) = q_s\,(I) + q_s\,(II) - q_s\,(I) \cdot q_s\,(II) \tag{25}$$

Then for identical units

$$q_s(A) = q_s(B) \tag{26}$$

$$q_s(A) = 2\,q_s - q_s^2 \tag{27}$$

$$q_o(A) = q_o(B) = q_o(I) \cdot q_o(II) = q_o^2 \tag{28}$$

Substituting equations (27) and (28) into equation (24) yields the probability of success directly as

$$P_S = (1 - q_o^2)^2 - (2\,q_s - q_s^2)^2 \tag{29}$$

$$P_S = (1 - q_o^2)^2 - \left[1 - (1 - 2\,q_s + q_s^2)\right]^2 \tag{30}$$

$$P_S = (1 - q_o^2)^2 - \left[1 - (1 - q_s)^2\right]^2 \tag{31}$$

The parallel series configuration, figure 12, can be solved in the same manner.

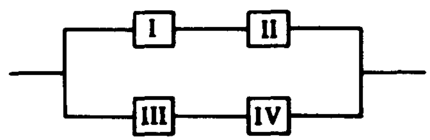


Figure 12 - Parallel Series

Following the same approach, equation (20) which was developed for the parallel network, the probability of success equation may be written as

$$P_S = (1 - q_s^2)^2 - (2\,q_o - q_o^2)^2 \tag{32}$$

$$P_S = (1 - q_s^2)^2 - \left[1 - (1 - q_o)^2\right]^2 \tag{33}$$

Table 3 - Redundant Network Equations

Configuration	P_S Equation	Minimizes
Parallel	$(1 - q_s)^2 - q_o^2$	Opens
Series	$(1 - q_o)^2 - q_s^2$	Shorts
Series Parallel	$(1 - q_o^2)^2 - [1 - (1 - q_s)^2]^2$	Opens
Parallel Series	$(1 - q_s^2)^2 - [1 - (1 - q_o)^2]^2$	Shorts

The series parallel network is sensitive to shorts. This can be shown by expanding equation (31).

$$P_S = (1 - 2\,q_o^2 + q_o^4) - (2\,q_s + q_s^2)^2 \tag{34}$$

dropping 3rd order terms and higher

$$P_S = 1 - 2\,q_o^2 - 4\,q_s^2 \tag{35}$$

If $q_o < 0.1$ and $q_s < 0.1$ the approximation is valid and equation (35) shows the effects of opens and shorts. Depending upon the relative frequency of occurrence of shorts and opens of a particular component one can choose the redundant configuration which yields optimum reliability.

The equation for the parallel network is an image of the series network. By putting in q_s for q_o and q_o for q_s in the parallel case, the series equation results. The same relation holds for the series parallel and parallel series. We will use this identity later in the report. If equations are developed for the parallel or series parallel network, they can be converted to the series or parallel series by the above process.

Time Dependent Probability of Success Equations

To derive the time dependent probability of success equations when utilizing components which fail in two modes, a brief discussion of the component's probability of success equations is necessary.

The probability of success equation derived for the parallel network is:

$$P_S = (1 - q_s)^2 - q_o^2 \tag{20}$$

To express the probability of success as a function of time, q_o and q_s should be explicit functions of time.

Multi-Mode Exponential Function

It is known that the components used in the network have two failure states, open or short. It is postulated that the distribution of failures due to open or shorts is exponential in nature. The form of the probability density function for the random events known as shorts and the random events known as opens will now be derived.

Two equations are used to derive these probability density functions. They are:

$$1 = P_s + \sum_{i=1}^{N} q_i \tag{36}$$

$$P_s = e^{-t/\bar{T}} \tag{37}$$

P_S is the probability of the component operating successfully, and has been shown to be an exponential function. q_i is the probability of failing in the ith mode. The sum of the probabilities of failing in all disjoint modes plus the probability of success (not failing in any of the above modes) must equal one.

$$P_S = 1 - \sum_{i=1}^{N} q_i \qquad \text{where } P_S \text{ is probability of success} \tag{36}$$

$$\sum_{i=1}^{N} q_i \qquad \text{is the exhaustive sum of probability of disjoint of failure events}$$

$$P_S = e^{-t/\overline{T}} \qquad \text{known} \tag{37}$$

$$\dot{P}_S = - \sum_{i=1}^{N} \dot{q}_i \qquad \text{time derivative} \tag{38}$$

$$\frac{e^{-t/\overline{T}}}{\overline{T}} = \sum_{i=1}^{N} \dot{q}_i \tag{39}$$

Consider two modes of failure namely that of open or short.

$$\frac{e^{-t/\overline{T}}}{\overline{T}} = \dot{q}_o + \dot{q}_s \qquad \begin{array}{l} q_o \text{ - probability of open} \\ q_s \text{ - probability of short} \end{array} \tag{40}$$

It is reasonable to postulate that $\dot{q}_o$ and $\dot{q}_s$ must be exponential since the left hand side of equation (40) is exponential. Therefore, $\dot{q}_o$ and $\dot{q}_s$ are assigned an exponential function with an arbitrary coefficient yet to be determined.

$$\dot{q}_o = \frac{A}{\overline{T}} e^{-t/\overline{T}} \tag{41}$$

$$\dot{q}_s = \frac{B}{\overline{T}} e^{-t/\overline{T}} \tag{42}$$

$$q_o = A (1 - e^{-t/\overline{T}}) \tag{43}$$

$$q_s = B\,(1 - e^{-t/\overline{T}}) \qquad (44)$$

$$P_S = 1 - q_o - q_s \qquad (45)$$

$$e^{-t/\overline{T}} = 1 - (A + B)\,(1 - e^{-t/\overline{T}}) \qquad (46)$$

$$A + B = 1 \qquad (47)$$

To determine A and B, consider N units are on life test. At some T, M units will have failed. These M failures are comprised of O opens and S shorts. If q is the probability of having a failure by time T

$$q = 1 - e^{-T/\overline{T}} = \frac{M}{N} \qquad (48)$$

$$q = q_o + q_s \qquad (49)$$

$$q_o = A\,(1 - e^{-T/\overline{T}}) \qquad (43)$$

Now q_o is the probability of having an open at time T. This is the ratio of total units on test to the units having opens at T.

$$q_o = \frac{O}{N} = A\,(1 - e^{-T/\overline{T}}) - A \cdot \frac{M}{N} \qquad (50)$$

$$A = \frac{O}{M} \qquad (51)$$

Therefore A is the ratio of units failed to units failed with open. B can be written as

$$B = \frac{S}{M} \qquad (52)$$

Given test data on a particular component A and B can be determined. This is discussed in the section on Quad Design.

Redundant Network Equations

The probability of success equations for the parallel and series networks are

$$P_S = (1 - q_s)^2 - q_o^{\,2} \qquad (20)$$

$$P_S = (1 - q_o^{\,2}) - q_s^{\,2} \qquad (25)$$

$$P_S = [1 - B(1 - e^{-t/\overline{T}})]^2 - A^2(1 - e^{-t/\overline{T}})^2 \quad (53)$$

$$P_S = [1 - A(1 - e^{-t/\overline{T}})]^2 - B^2(1 - e^{-t/\overline{T}})^2 \quad (54)$$

The probability of success equations for the series parallel and parallel series are

$$P_S = (1 - q_o^2)^2 - [1 - (1 - q_s)^2]^2 \quad (31)$$

$$P_S = (1 - q_s^2)^2 - [1 - (1 - q_o)^2]^2 \quad (33)$$

Once again direct substitution of q_s and q_o yields

$$P_S = [1 - A^2(1 - e^{-t/\overline{T}})^2]^2 - \left\{1 - [1 - B(1 - e^{-t/\overline{T}})]^2\right\}^2 \quad (55)$$

$$P_S = [1 - B^2(1 - e^{-t/\overline{T}})^2]^2 - \left\{1 - [1 - A(1 - e^{-t/\overline{T}})]^2\right\}^2 \quad (56)$$

Table 4. Time Dependent Redundant Network Equations

Network	Reliability Equation	Sensitivity
Parallel	$[1-B(1-e^{-t/\overline{T}})]^2-A^2(1-e^{-t/\overline{T}})^2$	Shorts
Series	$[1-A(1-e^{-t/\overline{T}})]^2-B^2(1-e^{-t/\overline{T}})^2$	Opens
Series Parallel	$[1-A^2(1-e^{-t/\overline{T}})^2]^2-\left\{1-[1-B(1-e^{-t/\overline{T}})]^2\right\}^2$	Shorts
Parallel Series	$[1-B^2(1-e^{-t/\overline{T}})^2]^2-\left\{1-[1-A(1-e^{-t/\overline{T}})]^2\right\}^2$	Opens

Figures 13 - 17 plot the relationship of probability of success versus time for the single unit, parallel, and the series parallel network. While the points are plotted to twice the mean life the useful portion is out to about 0.4 T.

Figure 13 shows that the parallel network is superior to the series parallel when only opens occur, A = 1. In these figures, the single unit is represented by a straight line since $e^{-t/T}$ on semi-log paper is a linear function.

Figure 14, A = 0.9, B = 0.1, shows the series parallel unit slightly better than the parallel unit. As the probability of shorts increases the series parallel unit becomes more reliable than the parallel unit. The series parallel unit reaches its most reliable mode when A = 0.7 and B = 0.3. These values are taken from the curves rather than differentiating equation (55) with respect to A and setting the result to zero and solving for A which gives max P_s. When A = 0.5, B = 0.5 the parallel unit becomes equivalent to a single unit.

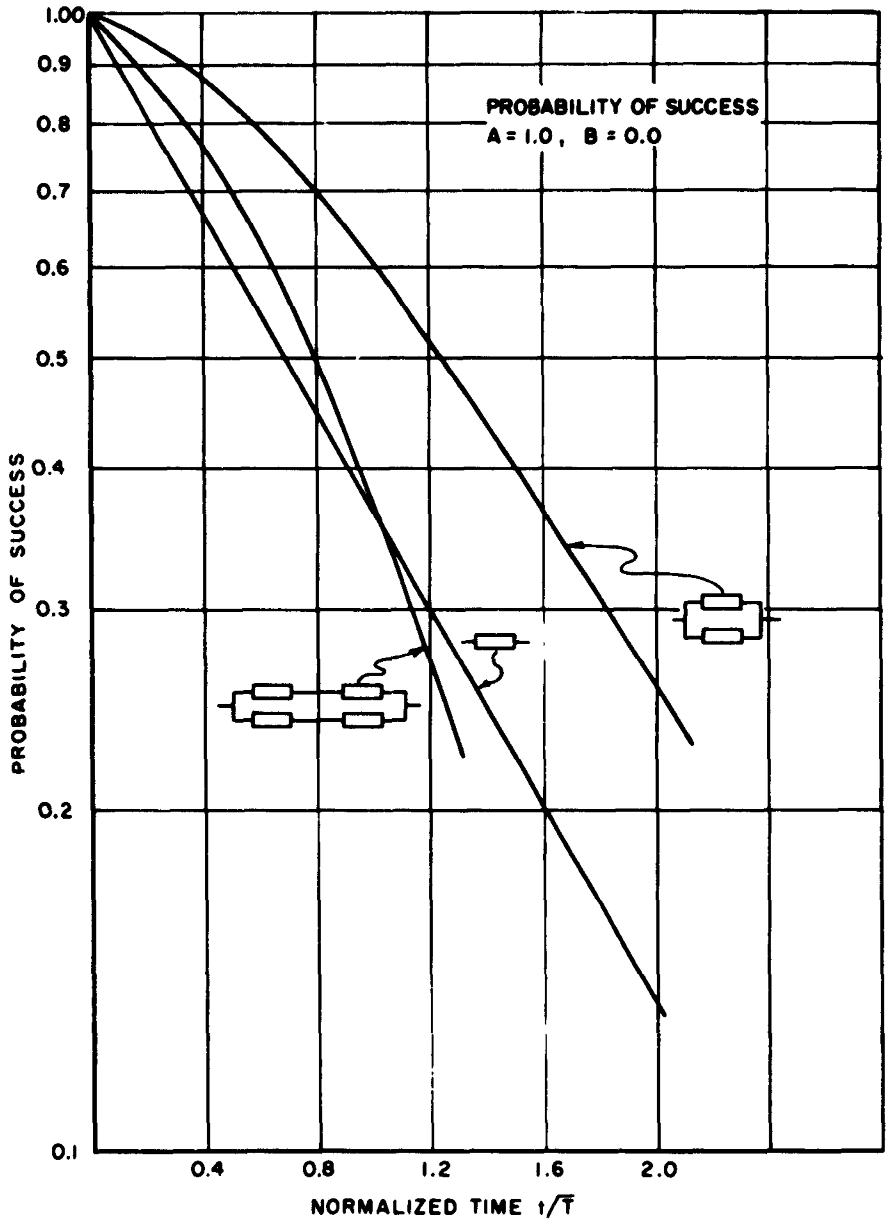


Figure 13. Probability of Success versus Time, A = 1.0

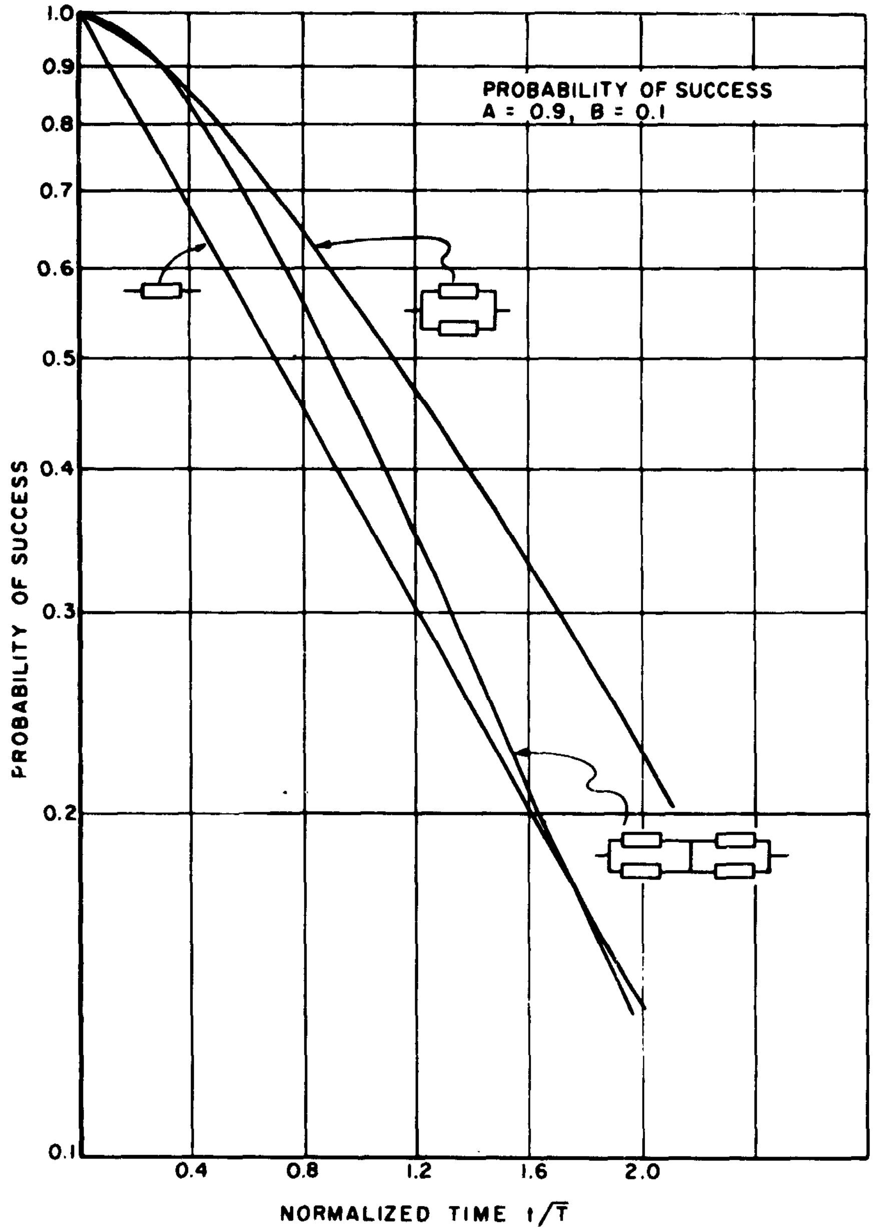


Figure 14. Probability of Success versus Time, A = 0.9

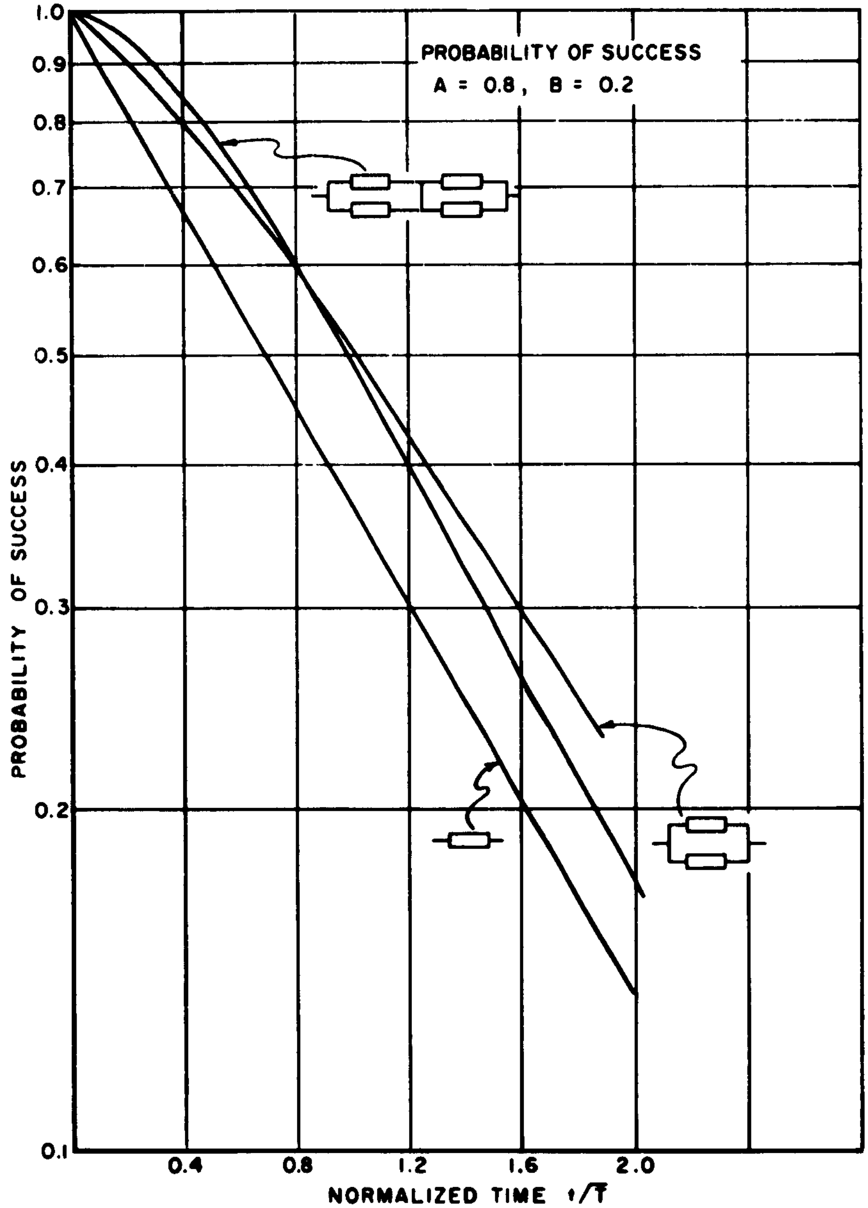


Figure 15. Probability of Success versus Time, A = 0.8

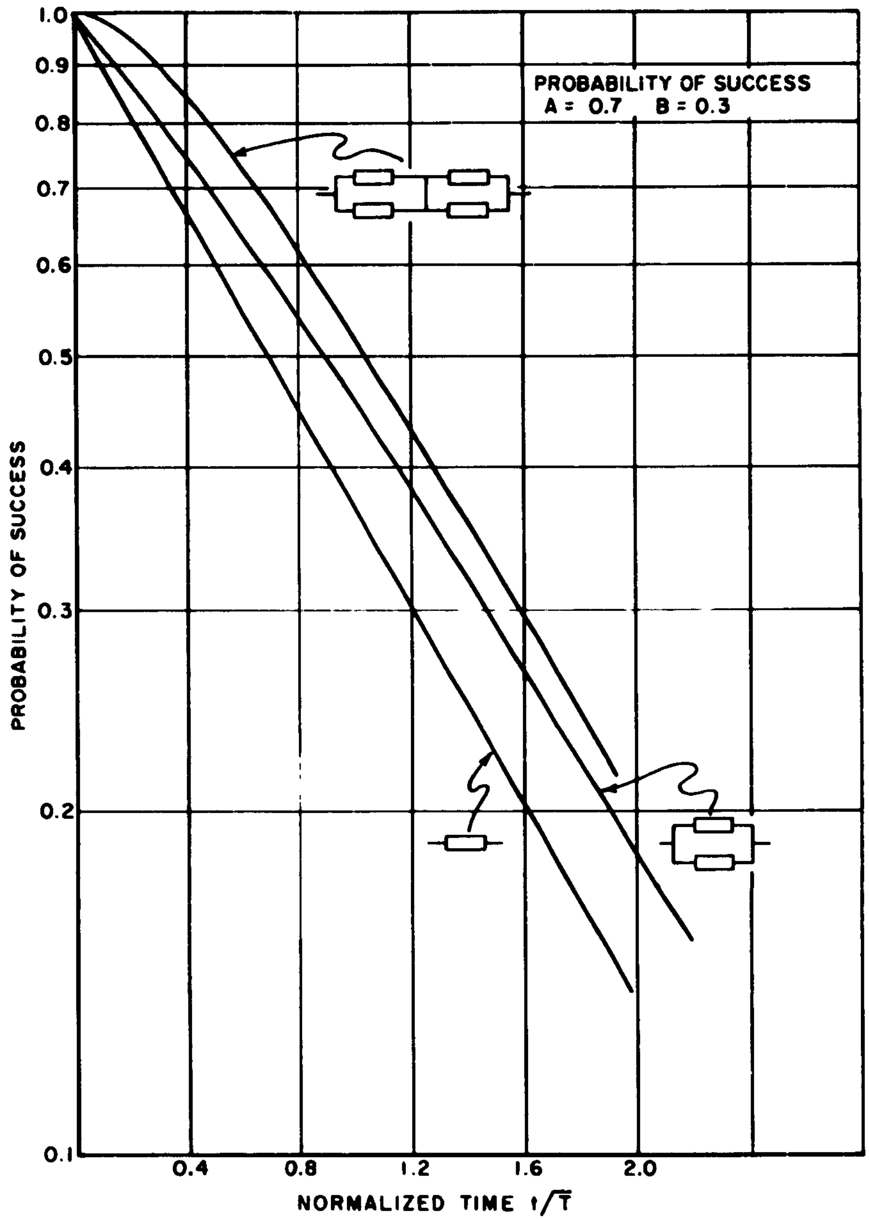


Figure 16. Probability of Success versus Time, A = 0.7

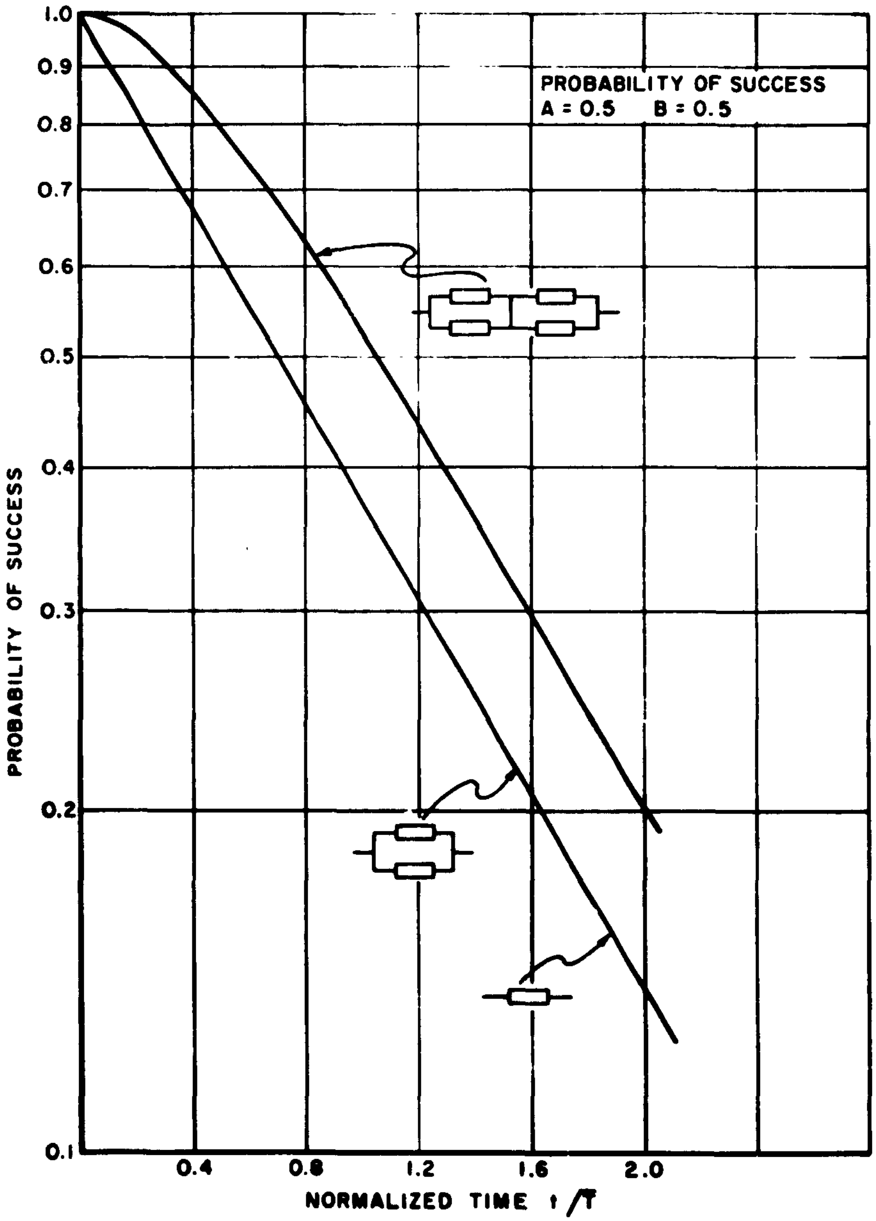


Figure 17. Probability of Success versus Time, A = 0.5

QUAD NETWORK THEORY

The purpose of this section is the orderly development of the probability of success equations of the Quad. The quad network is a network of four blocks arranged in a parallel-series or series- parallel mode. These blocks may be identical or non-identical, one-element or multi-element. Figure 18 shows the two basic quad networks within quad networks

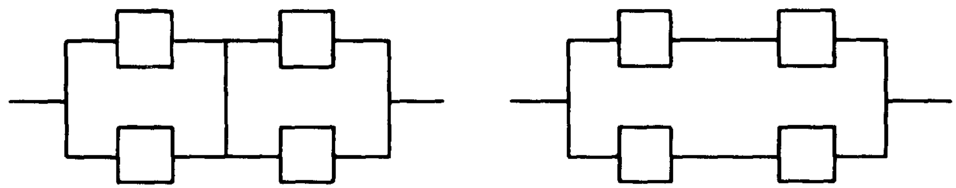


Figure 18. Quad Networks

A network has been designed and built both in an unquaded and in a quaded state to show the results of quading. This network is shown in figure 19.

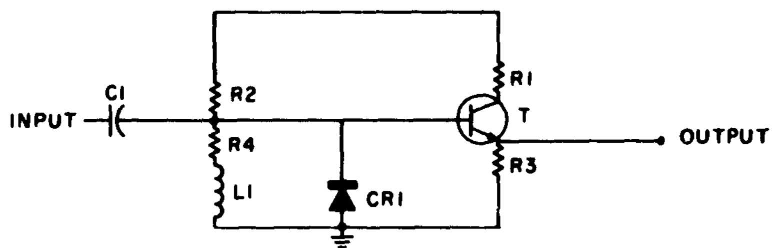


Figure 19. Impedance Transformer Network

We desire to make this network as reliable as possible. Assume that this network is a very critical link in a satellite transmitter. One way to increase reliability would be to parallel two identical networks, figure 20. This technique will be analyzed and compared to the quad.

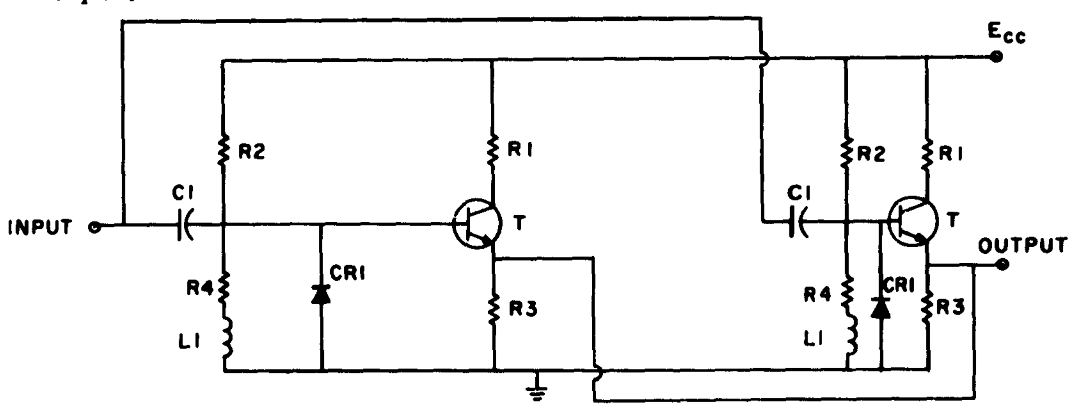


Figure 20. Parallel Transistor Network

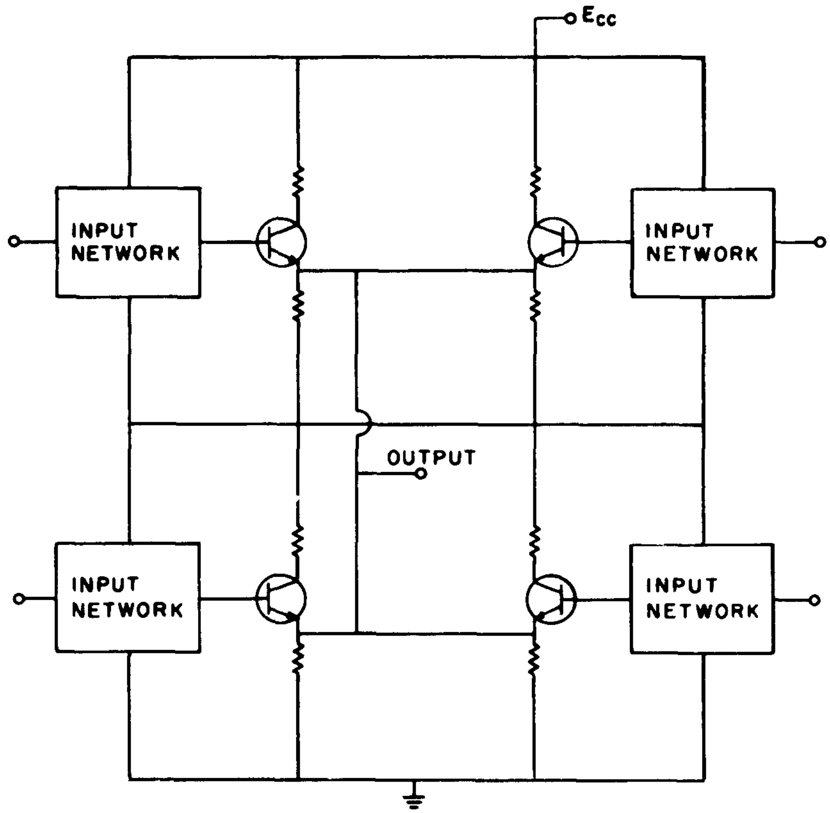


Figure 21. Series Parallel Transistor Network

A continuation of this idea would be to make a series parallel arrangement, figure 21.

A less obvious design would be to redund the components themselves. For example, the transistor can fail in open or short. The quad approach would yield figure 22.

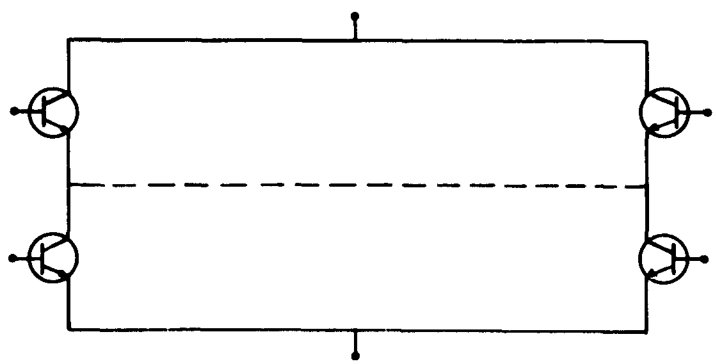


Figure 22. Transistor Quad

Failure mode analysis will indicate whether the dashed line should remain or be removed. Before a circuit can be quaded on a component level knowledge of the failure modes of the components must be possessed. Knowing the A and B figures we can determine the most reliable network configuration for a particular component. For any component we have the choice of a parallel, series, series parallel, or parallel series network. Table 5 is a tabulation of failure mode data for typical electronic components. The data is used to illustrate the method developed in this report. To design a quad using particular components, if it is at all possible, use data based upon life tests on that component. This data would be far superior to table 5. The relative size of A and B is the most important.

Going back to figure 22 the question was raised as to the line connecting the parallel branches. Table 5 gives an A of 0.84 for the transistors tested. Figure 15 shows that the series-parallel network is more reliable than the parallel network. The concept of images as developed earlier in the report coupled with table 5 tells us that the parallel series is less reliable than the series parallel. Therefore, the line remains in.

Resistors and inductors are assumed to fail only in open, never in short. Capacitors are assumed to fail only in short, never in open. Figure 13, plot of reliability of several networks for A = 1.0, B = 0, shows that the parallel network is more reliable than the single unit or the series parallel for the resistor. This implies that for the capacitor the series network is more reliable than the single unit or the parallel series.

To quad the components of figure 19 using the data on table 5, figure 23 results.

Table 5 Failure Data

Component	Number Tested ($\times 10^3$)	Failures		$\frac{N(S)}{N(F)}$	$\frac{N(O)}{N(F)}$	Failure* Rate ($\times 10^{-6}$)
		Short	Open			
Transistors	98	79	423	.16	.84	0.03
Diodes	447	67	348	.16	.84	0.2
Capacitors	215	3	0	1	0	0.09
Resistors	500	0	0	--	--	0.014
Inductor	--	--	--	--	--	0.001

$$A = \frac{N(O)}{N(F)} \qquad B = \frac{N(S)}{N(F)}$$

*The failure rate information was taken from ASD Report 61-580.

We desire to derive an equation which relates probability of success as a function of time for figure 23. Figure 24 represents figure 23 where blocks I and III are resistor networks 3 and 4. Block II is the remainder.

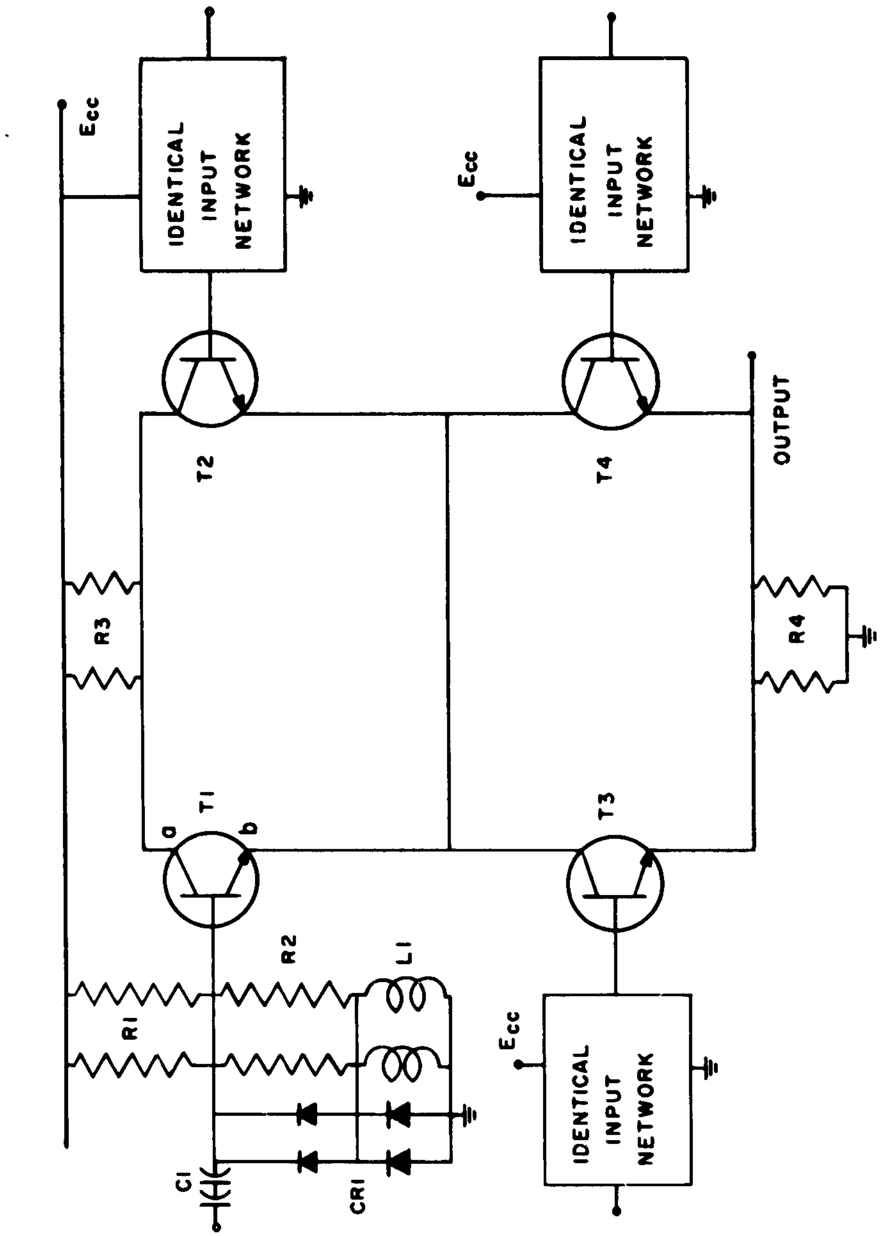


Figure 23. Quad Network

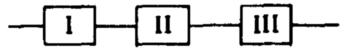


Figure 24 - Simplified Network

The probability of success of figure 24 is

$$P_S = P(I) \cdot P(II) \cdot P(III) \tag{57}$$

$$P(I) = P(III) = 1 - (1-e^{-t/TR})^2 \tag{58}$$

$$P(I) = P(III) = 2\,e^{-t/TR} - e^{-2t/TR} \tag{59}$$

Equation (58) is the probability of success of a parallel network, equation (53), with $A = 1, B = O$.

Block II is the transistor quad with the necessary input and bias networks, see figures 22 and 23. The first problem is to derive the probability that one transistor ensemble will open and the probability that one transistor ensemble will short. Each ensemble consists of:

Item	Quantity	Components
Transistors	1	1
Parallel resistor network	2	2
Parallel inductor network	1	2
Series capacitor network	1	2
Series parallel diode network	1	4

The events which will cause an apparent short across terminal a-b, transistor 1 in figure 23 will be tabularized.

Table 6. Events Causing Short

Events	Symbol	Probability	Basic Equation
Transistor shorts	$q_s(T)$	$B_T(1 - e^{-t/TT})$	44
Capacitor network shorts	$q_s(C_1)$	$(1 - e^{-t/TC})^2$	54
Diode network shorts	$q_s(CR)$	$(1 - [1 - B_{CR}(1 - e^{-t/TCR})]^2)^2$	56

For the ensemble to fail in open one of the following events must occur.

Table 7. Events Causing Open

Events	Symbol	Probability	Basic Equation
R_1 network opens	$q_o(R_1)$	$(1 - e^{-t/TR})^2$	53
R_2 network opens	$q_o(R_2)$	$(1 - e^{-t/TR})^2$	53
Inductor network opens	$q_o(L)$	$(1 - e^{-t/TL})^2$	53
Transistor Opens	$q_o(T)$	$A_T(1 - e^{-t/TT})$	43
Diode network opens	$q_o(C_R)$	$1 - [1-A^2{}_{CR}(1 - e^{-t/TCR})^2]^2$	55

By listing the short events the probability, QS, that one transistor ensemble will short is calculated by summing the probability of occurrence of all disjoint short events.

$$\begin{aligned} QS = {} & q_s(T) + q_s(C_1) + q_s(T)\, q_s(C_1) \\ & -q_s(C_R)\, q_s(C_1) - q_s(CR)\, q_s(T) \\ & +q_s(CR)\, q_s(C_1)\, q_s(T) \end{aligned} \tag{60}$$

An alternate method is to calculate the probability, P_S, that the ensemble will not short. P_s is the product of the individual "not probabilities". P_s is the probability that the capacitor network will not short, that the transistor will not short, and that diode network will not short.

$$P_s = [1 - q_s(T)] \cdot [1 - q_s(C_1)] \cdot [1 - q_s(CR)] \tag{61}$$

The probability, QS, of short is

$$QS = 1 - P_s = 1-[1-q_s(T)] \cdot [1-q_s(C_1)] \cdot [1-q_s(CR)] \tag{62}$$

The reader can expand 62 to show the identity with 60.

To sum up all the probabilities in table 7 as we did with table 6 would yield an equation similar to equation (60) but with 31 terms. Using the method to develop equation (62) the probability of the transistor ensemble, QO, opening is

$$QO = 1-[1-q_o(T)] \cdot [1-q_o(L)] \cdot [1-q_o(CR)] \cdot [1-q_o(R)]^2 \tag{63}$$

A series parallel network with known probabilities of opening and shorting QO and QS can be constructed. See figure 25.

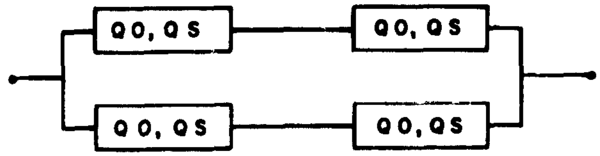


Figure 25. Transistor Ensemble

This looks very much the same as the network in figure 11 which had a probability of short, q_s, and probability of open, q_o, and a probability of success equation

$$P_s = (1 - q_o^2)^2 - [1 - (1 - q_s)^2]^2 \qquad (31)$$

This equation cannot be used on figure 25. The problem is that in equation (31), q_o and q_s are mutually exclusive events. Both a short and an open cannot occur in the same element.

If the blocks in figure 25 represented just transistors then equation (31) is valid. If QS is probability of short and QO is probability of open the general equation for the term relating the probability of either event or both occurring in one block is

$$Q(O + S) = QO + QS - Q\,(O, S) \qquad (64)$$

Now Q(O, S) is a joint or conditional probability. In the case of mutually exclusive events this term is zero. Whether the events are mutually exclusive or not Q(O + S) must be between zero and one.

$$O < Q\,(O + S) = QO + QS - Q\,(O, S) < 1 \qquad (65)$$

If it can be shown that the sum of QO and QS for figure 25 exceeds one, then the events must not be mutually exclusive.

To show that QS approaches 1 as time becomes quite large, it can be shown that one term of equation (62) approaches zero. For example

$$(1 - q_s\,(C_1)) = 1 - (1 - e^{-t/TC})^2 \rightarrow o \text{ as } t \rightarrow \infty \qquad (66)$$

Therefore QS $\rightarrow$ 1 as $t \rightarrow \infty$

Similarly QO $\rightarrow$ 1 as $t \rightarrow \infty$

Therefore QS + QO $\rightarrow$ 2 as $t \rightarrow \infty$

Equation (31) can be modified to account for the fact that q_o and q_s are not mutually exclusive. This is done by writing the right hand side of equation (31) thus:

$$P_s = 1 - (2q_o^2 - q_o^4) - [1 - (1 - q_s)^2]^2 \qquad (67)$$

The probability of failure due to opens, QOO, is

$$QOO = (2\,q_o^2 - q_o^4) \qquad (68)$$

The probability of failure due to shorts, QSS, is

$$QSS = [1 - (1 - q_s)^2]^2 \tag{69}$$

Since QSS and QOO are not mutually exclusive, the probability of failure of the quad can be written as

$$Q = QOO + QSS - QOO \cdot QSS \tag{70}$$

The probability of success is then

$$P = 1 - Q \tag{71}$$

$$P = 1 - QOO - QSS + QOO \cdot QSS \tag{72}$$

$$P = 1 - (2q_o^2 - q_o^4) - [1-(1 - q_s)^2]^2 + (2q_o^2 - q_o^4)[1 - (1 - q_s)^2]^2 \tag{73}$$

$$= (1-q_o^2)^2 + (2q_o^2-q_o^4 - 1) \cdot [1-(1-q_s)^2]^2 \tag{74}$$

$$= (1-q_o^2)^2 - (1 - q_o^2)^2 \cdot [1-(1 - q_s)^2]^2 \tag{75}$$

$$P = \left\{1 - [1-(1-q_s)^2]^2\right\}(1-q_o^2)^2 \tag{76}$$

Equation (76) is used to evaluate the reliability of figure 25 which is block II of figure 24.

CONCLUSIONS

The design engineer can use these methods to predicate the reliability of redundant networks. He can choose the network which yields the level of reliability needed without undue complexity. The design engineer would not design a two kilowatt transmitter when 500 watts will suffice and he would not design equipment with a higher reliability than is necessary.

To improve the reliability of the large airborne computer mentioned earlier, not every circuit should be made quad redundant. Those circuits, which by their nature lend themselves to redundancy, should be redunded first; the resulting reliability calculated, and further work done as necessary.

Figure 26 is the probability of success plot of the four networks discussed in the report. The results are what was expected. The quad is best followed by the series-parallel, parallel, and lastly the non-redundant circuit. What purpose is this report if the results were known beforehand? Previous to this, no one knew how much better the quad was over the other circuits. It is this ability of being able to predicate the improvement in reliability which makes this work worth the effort. Furthermore, the quad has been analyzed rigorously, knowing that components can fail in two modes as opposed to failing in one as has been the case in previous investigations.

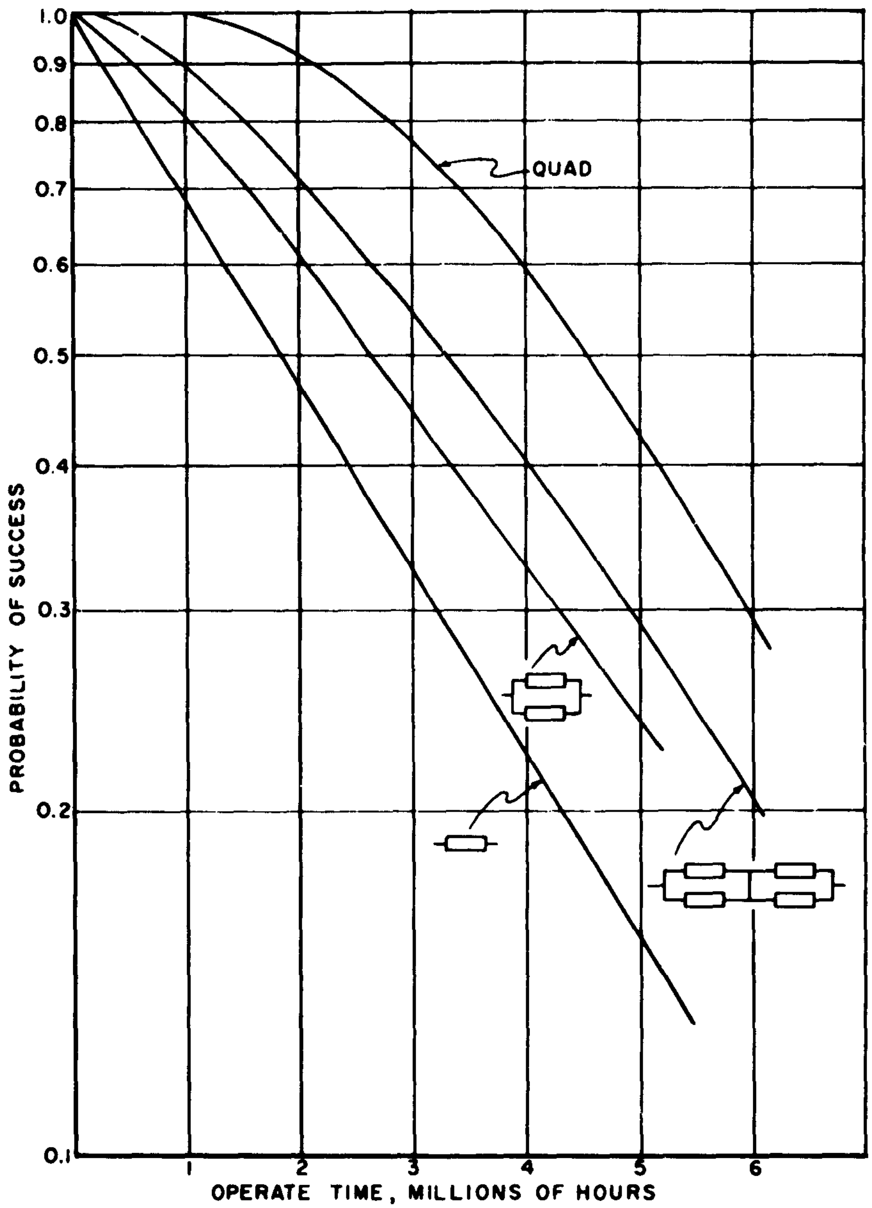


Figure 26. Redundant Network Probability of Success versus Time

Figure 27 shows the relative effects of redundancy upon large electronic systems. The 1000 and 5000 curves refer to a system employing 1000 or 5000 transistor networks. Each transistor network is comprised of the same elements as in the impedance transformer. The curves demonstrate the effect of multiplying a number less than one a large number of times. The probability of success of the 1000 unit system is the probability of success of the basic unit raised to the 1000th power. The quad becomes very superior to the non-redundant network while the series parallel unit lies somewhere in between. The 5000 unit is not an unrealistic size as the Air Force has an airborne computer somewhat larger than this presently in production. The decision to use the series parallel or quad for a particular network depends upon the relative difficulty in designing and fabricating that network.

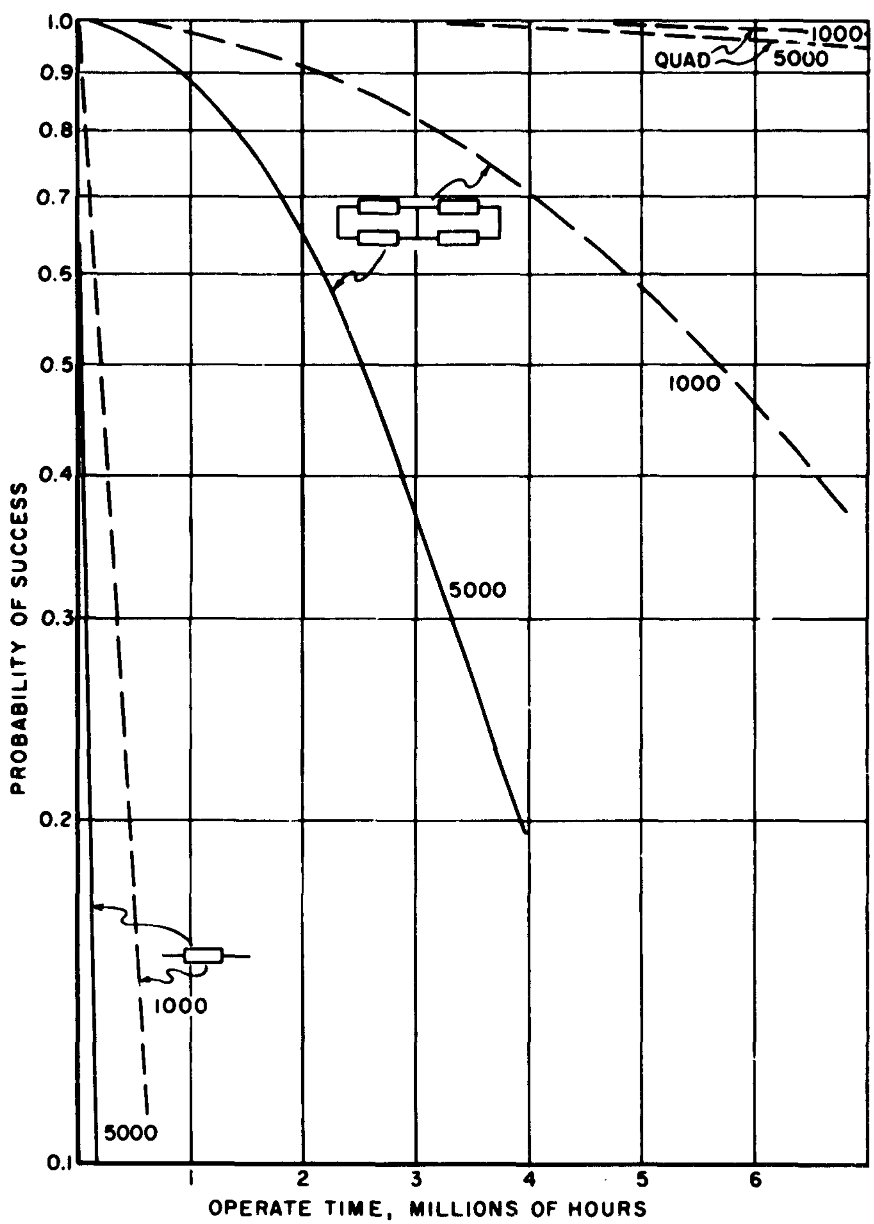


Figure 27. Probability of Success versus Time 1000 and 5000 Serial Units

RECOMMENDED AREAS OF FUTURE INVESTIGATION

The application of redundancy to electronic design is limited by our ability to design redundant networks. The input-output characteristics may be such that active redundancy cannot be used. However, most pulse or digital circuits are easily adapted to redundant configuration. Robert Lyons discusses redundant design in his report "The Use of Triple Modular Redundancy to Improve Computer Reliability." While triple modular redundancy is not the same as component redundancy, Lyons' recommendations are similar to this author's. Namely, that the design engineer must design reliability into the network primarily by using designs based upon probability theory. Pulse and digital circuits must be investigated with this concept in mind.

A second area of investigation concerns the paralleling and serializing of electronic components. In building the quad it was found that the needed value of inductance of two paralleled inductors could be had when the mutual inductance was additive. If a coil then failed in open, the only mode of failure, the change in inductance was not excessive. For example, the average value of the eight coils was 172 microhenrys. The average value of the paralleled inductors was 120 microhenrys. The change from 120 to 170 microhenrys was not so great as to degrade performance. Along this same line, it might be necessary to use three resistors rather than two in order to reduce the net change in resistance when an open occurs. Investigation along these lines would be very fruitful.

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